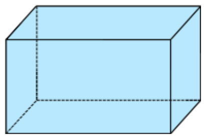
Precalc Warm Up # 13-2

Daisy skis 800 m to a point P, due East of her cabin at point O, then 500 m due North where she reaches a cliff, point Q. She then jumps straight down 80 m to point R. (She's practicing for the Olympics.)

1. Draw a vector diagram showing the vectors OP, PQ, and QR



2. Find to the nearest 10th. a. |OQ| b. |OR|

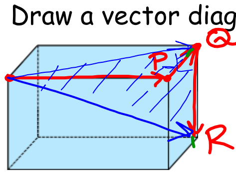
Do you notice a shortcut to find the diagonal in the box?

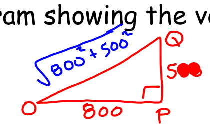
Precalc Warm Up # 12-2

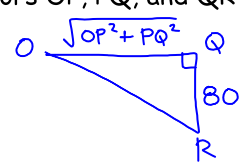
Daisy skis 800 m to a point P, due East of her cabin at point O, then 500 m due North where she reaches a cliff, point Q. She then jumps straight down 80 m to point R. (She's practicing for the Olympics.)

$|OP| = 800 \quad |PQ| = 500 \quad |QR| = 80$

1. Draw a vector diagram showing the vectors $\overrightarrow{OP}$, $\overrightarrow{PQ}$, and $\overrightarrow{QR}$







≈ 943.4

2. Find to the nearest 10th. a. $|\overrightarrow{OQ}|$ b. $|\overrightarrow{OR}|$

$$OR^2 = \left(\sqrt{OP^2+PQ^2}\right)^2 + QR^2$$

$$\sqrt{OR^2} = \sqrt{OP^2+PQ^2+QR^2}$$

$$OR = \sqrt{800^2+500^2+80^2}$$

$$\approx 946.8$$

Do you notice a shortcut to find the diagonal in the box?

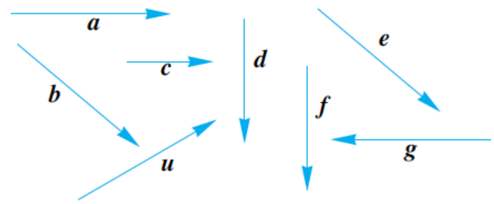
HW Questions, p. 413

2. The vector represents a velocity of 20 m/s due West. Represent the following vectors:

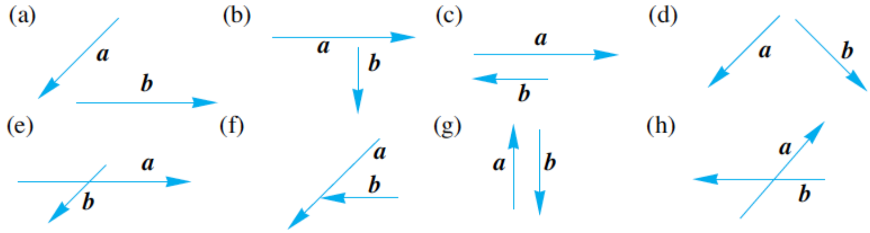
(a) 20 m/s due East
(b) 40 m/s due West
(c) 60 m/s due East
(d) 40 m/s due NE

3. State which of the vectors shown

(a) have the same magnitude.
(b) are in the same direction.
(c) are in the opposite direction.
(d) are equal.
(e) are parallel.

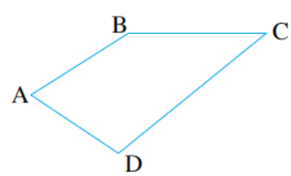


4. For each of the following pairs of vectors, find $\boldsymbol{a} + \boldsymbol{b}$.

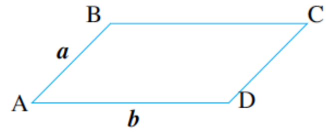


5. For the shape shown, find a single vector which is equal to

(a) **AB** + **BC**
(b) **AD** + **DB**
(c) **AC** + **CD**
(d) **BC** + **CD** + **DA**
(e) **CD** + **DA** + **AB** + **BC**



6. Consider the parralelogram shown alongside.
Which of the following statements are true?



(a) $\mathbf{AB} = \mathbf{DC}$ (b) $|\boldsymbol{a}| = |\boldsymbol{b}|$

(c) $\mathbf{BC} = \boldsymbol{b}$ (d) $|\mathbf{AC} + \mathbf{CD}| = |\boldsymbol{b}|$

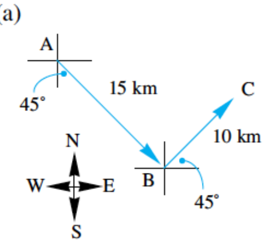
(e) $\mathbf{AD} = \mathbf{CB}$

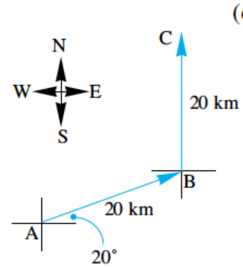
7. For each of the following

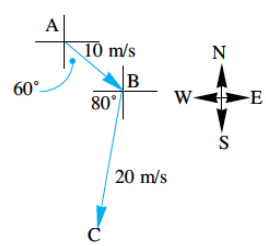
i. complete the diagram by drawing the vector $\mathbf{AB} + \mathbf{BC}$.

ii. find $|\mathbf{AB} + \mathbf{BC}|$.

(a) (b) (c)





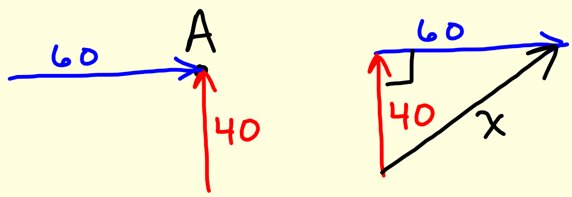


8. Two forces, one of 40 Newtons acting in a Northerly direction and one of 60 Newtons acting in an Easterly direction, are applied at a point A. Draw a vector diagram representing the forces. What is the resulting force at A?

9. Two trucks, on opposite sides of a river, are used to pull a barge along a straight river. They are connected to the barge at one point by ropes of equal length. The angle between the two ropes is 50°. Each truck is pulling with a force of 1500 Newtons.

(a) Draw a vector diagram representing this situation.

(b) Find the magnitude and direction of the force acting on the barge.



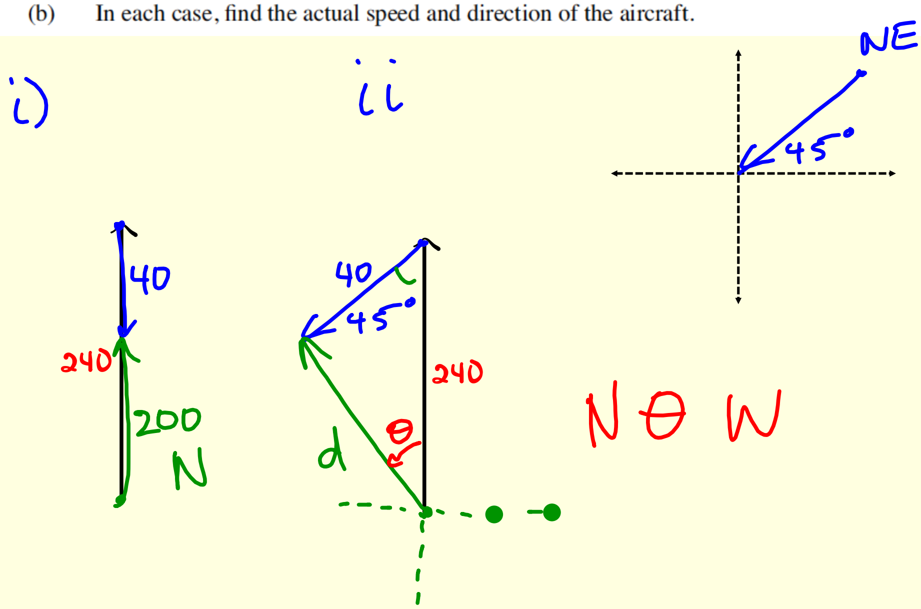
10. An aircraft is flying at 240 km/h in a Northerly direction when it encounters a 40 km/h wind from

i. the North.

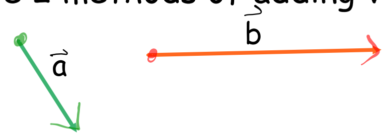
ii. the North-East.

(a) Draw a vector diagram representing these situations.

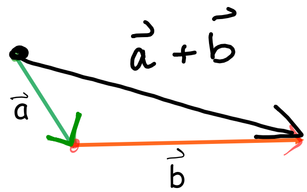
(b) In each case, find the actual speed and direction of the aircraft.



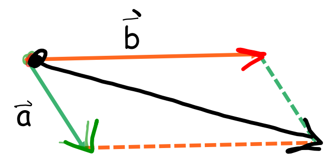
There are 2 methods of adding vectors:



The triangle method that we saw yesterday $\vec{a} + \vec{b} = \vec{b} + \vec{a}$

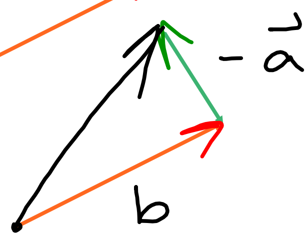


The parallelogram method

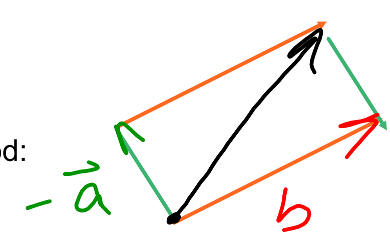


Subtracting: b - a

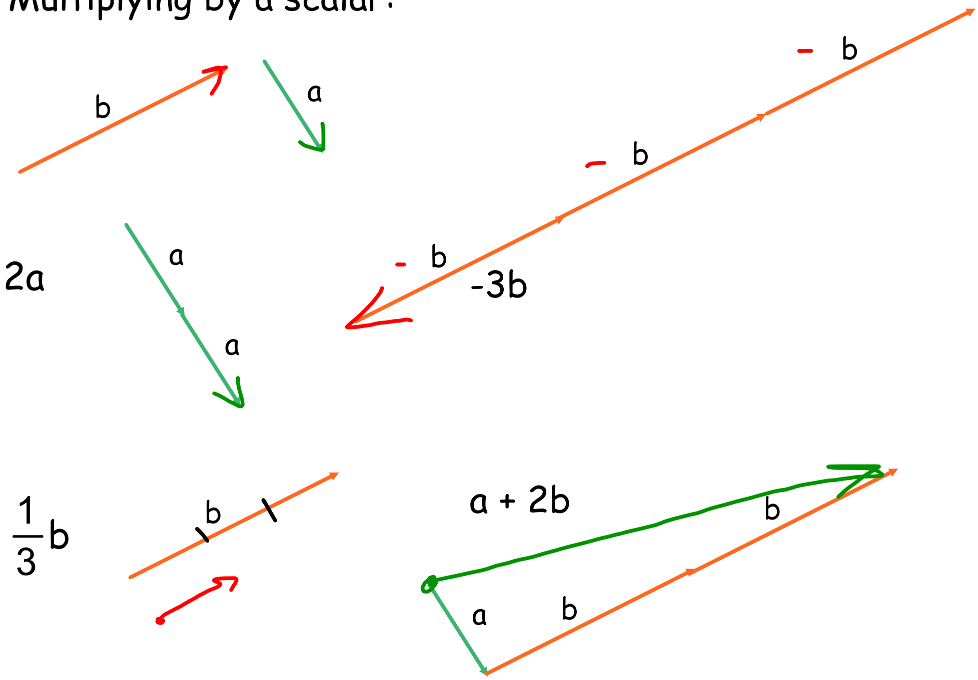
Triangle method:



Parallelogram method:



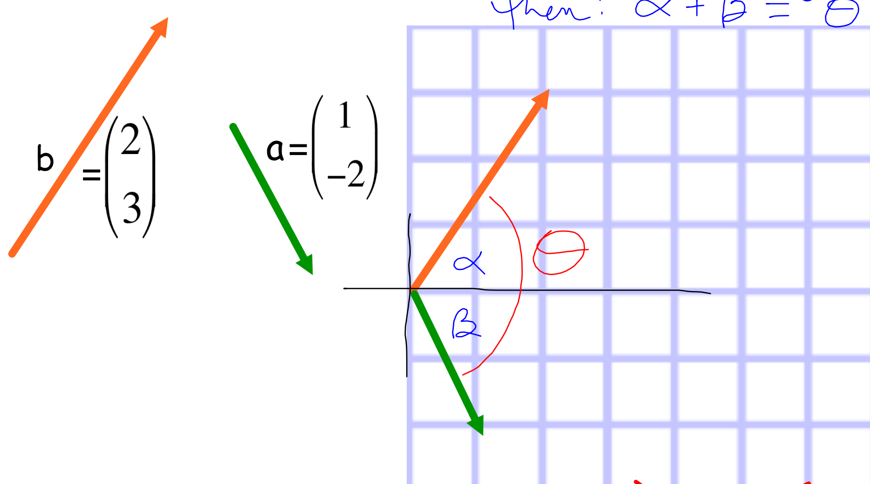
Multiplying by a scalar:



Find the angle between the vectors.

easiest: use Rt Δ Trig.

then: $\alpha + \beta = \theta$



Does it matter if they are joined ~~head to tail~~ or tail to tail?

Yes

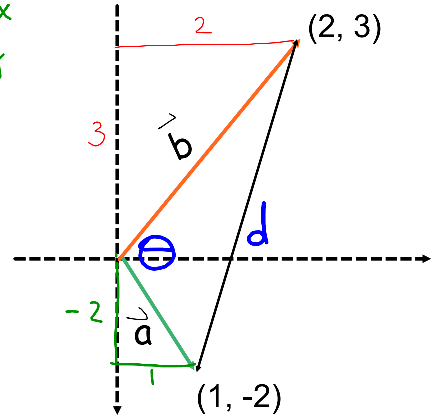
$\vec{a} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \begin{matrix} x \\ y \end{matrix}$ $\qquad$ $\vec{b} = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{matrix} x \\ y \end{matrix}$

$|\vec{a}| = \sqrt{1^2 + (-2)^2}$

$= \sqrt{5}$

$|\vec{b}| = \sqrt{2^2 + 3^2}$

$= \sqrt{13}$



$d = \sqrt{(x-x)^2 + (y-y)^2}$

$d = \sqrt{(2-1)^2 + (3+2)^2}$

$d = \sqrt{26}$

$$\theta = \cos^{-1}\left(\frac{(\sqrt{5})^2 + (\sqrt{13})^2 - (\sqrt{26})^2}{2\sqrt{5}\sqrt{13}}\right)$$

$$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{65}}\right) \approx 119.7^\circ$$

The advantages of the first method?

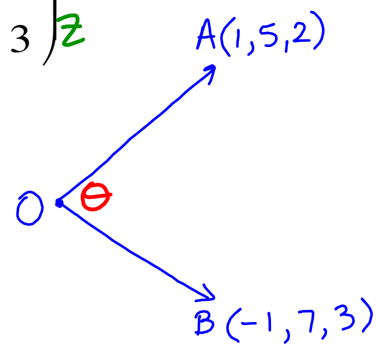
easier!

The advantages of the second method?

It will work better in 3-D space, as you will see!!!

Find the angle made between these 2 vectors.

$$\vec{A} = \begin{array}{c|c} \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array} \qquad \vec{B} = \begin{array}{c|c} \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix} & \begin{matrix} x \\ y \\ z \end{matrix} \end{array}$$



① Draw vectors tail to tail

② Find:

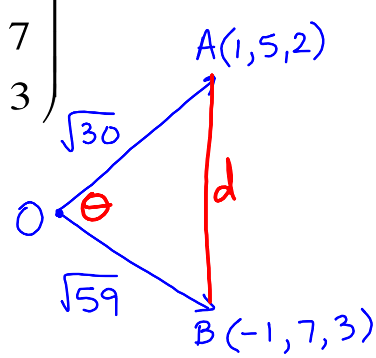
$|\vec{A}|$ & $|\vec{B}|$

$$|\vec{A}| = \sqrt{1^2 + 5^2 + 2^2} = \sqrt{30}$$

$$|\vec{B}| = \sqrt{(-1)^2 + 7^2 + 3^2} = \sqrt{59}$$

Find the angle made between these 2 vectors.

$$\vec{A} = \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} \qquad \vec{B} = \begin{pmatrix} -1 \\ 7 \\ 3 \end{pmatrix}$$



③ Find 3rd side of △

$$d = \sqrt{(x-x)^2 + (y-y)^2 + (z-z)^2}$$

④ Use Law of Cosines.

"

$$d = \sqrt{(1+1)^2 + (5-7)^2 + (2-3)^2} = 3$$

$$\theta = \cos^{-1}\left(\frac{(\sqrt{30})^2 + (\sqrt{59})^2 - 3^2}{2\sqrt{30}\sqrt{59}}\right) \approx 18.1^\circ$$

A little vector algebra

Easy....

AB + BC + DE + CD

Harder.....

2AB + 4AC + 4BA + 6CD + 2DA + 2DC

$2[\quad]$

A little vector algebra

Easy....

AB + BC + DE + CD

$AC + \underbrace{CD + DE} = AE$

Harder.....

2AB + 4AC + 4BA + 6CD + 2DA + 2DC

$2[AB + 2AC + 2BA + 3CD + DA + DC]$

$\underbrace{AB + BA}_{0} + BA \qquad 2CD + \underbrace{CD + DC}_{0}$

$2[2AC + BA + 2CD + DA]$

$\underbrace{2AC + 2CD}_{2AD}$

$2[\underbrace{2AD}_{\text{split}} + BA + DA]$

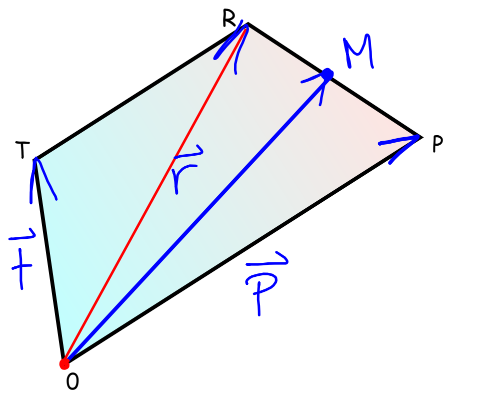
$2[BA + AD + \underbrace{AD + DA}_{0}]$

$\boxed{2BD}$

a little vector geometry...

In quadrilateral OTRP
$\overrightarrow{OT} = \vec{t}$
$\overrightarrow{OP} = \vec{p}$
$\overrightarrow{OR} = \vec{r}$

Find, in terms or t, p, and r an expression for



a. $\overrightarrow{TP}$

$\overrightarrow{TO} + \overrightarrow{OP}$

$-\vec{t} + \vec{p}$

b. $\overrightarrow{RT}$

$-\vec{r} + \vec{t}$

c. the midpoint of $\overrightarrow{RP}$ relative to O

$$\overrightarrow{OM} = \overrightarrow{OP} + \overrightarrow{PM}$$
$$= \vec{p} + \tfrac{1}{2}(\overrightarrow{PR})$$
$$= \vec{p} + \tfrac{1}{2}(-\vec{p} + \vec{r})$$
$$= \tfrac{1}{2}\vec{p} + \tfrac{1}{2}\vec{r}$$

HW: p. 421 # 1-4, 7, 8

Remember:

When the book writes **AB**,

it means $\overrightarrow{AB}$.